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A construction of commutative D-branes from lower-dimensional non-BPS D-branes

Seiji Terashima

Department of Physics, Faculty of Science
University of Tokyo, Tokyo 113-0033, Japan
E-mail: `seiji@hep-th.phys.s.u-tokyo.ac.jp`

Abstract: We construct an exact soliton, which represents a BPS Dp-brane, in a boundary string field theory action of infinitely many non-BPS D$(p-1)$-branes. Furthermore, we show that this soliton can be regarded as an exact soliton in the full string field theory. The world-volume theory of the BPS Dp-brane constructed in this way becomes usual gauge theory on commutative $\mathbb{R}^{p+1}$, instead of non-commutative plane. We also construct a Dp-brane from non-BPS D$(p-n)$-branes by the ABS like configuration. We confirm that the Dp-brane has correct RR charges and the tension.

Keywords: Solitons Monopoles and Instantons, Superstrings and Heterotic Strings, D-branes, Non-Commutative Geometry.

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Contents

1. Introduction

The idea of construction of a Dp-brane from infinitely many lower-dimensional D$(p-2)$-branes has been developed in the context of the Matrix theory [1]–[6]. Though this is very interesting idea, it was known that the Dp-brane which is constructed from BPS D$(p-2)$-branes has charge of the BPS D$(p-2)$-branes because the lower-dimensional brane charges should be conserved. This means that the world-volume of the BPS Dp-brane becomes non-commutative space [7]–[11]. Therefore it is interesting to construct a pure Dp-brane with commutative world-volume from lower-dimensional D-branes.

On the other hand, some properties of the system containing tachyon in string theories have been understood [12, 13, 14]. In particular, the non-BPS branes [15]–[19] and the brane-antibrane systems [20, 21, 22] have been studied. Furthermore, the exact solitons has been found in the effective non-commutative world-volume theories of unstable D-branes [23]–[28].[1] Here the solitons represent lower-dimensional D-branes. This effective theories can be regarded as the tree level actions of string field

[1] This construction can be viewed as a condensation of the tachyon which makes the lower-dimensional unstable $\infty - 1$ D$(p-2)$-branes disappear. A remaining D$(p-2)$-brane corresponds to the soliton solution in the non-commutative world-volume theory of the unstable Dp-brane because the infinitely many unstable D$(p-2)$-branes constitute the unstable Dp-brane. For more details, see appendix A.

theories, then, this solitons can be regarded as the exact solutions of equations of motion of string field theories. Recently, even for the case without non-commutativity, the exact solution has been found in the world-volume of the unstable D-branes using the boundary string filed theory (BSFT) [29]–[33], which was first formulated by Witten [34].

Then it is natural to try to construct a BPS D-brane from lower-dimensional unstable non-BPS D-branes which have no conserved charges. Although some attempts to do this was made in [35], the result is not clear, especially for the brane tension, since the BSFT action for the non-BPS brane had not been obtained and the actions used in [35] are not considered to be exact.

In this paper we construct an exact soliton, which represents a BPS Dp-brane, in the effective world-volume theory of infinitely many non-BPS D$(p-1)$-branes. This world volume theory is obtained from BSFT and the T-duality. In this paper we call a solution of the equations of motion the soliton although the solution is not localized in the world volume of the non-BPS D$(p-1)$-branes. This is because the solution is localized in the T-dual picture. Moreover, we can show that the solution remains intact even if we include the terms containing many commutators in the action. These terms should be included into the action when we consider the full string field theory action. Therefore, this soliton can be regarded as an exact soliton in the string field theory, as in the construction of the non-commutative soliton [28], where it is argued that derivative corrections, which correspond to higher commutator terms in our case, can be ignored.

The world-volume theory of the BPS Dp-brane constructed in this way becomes usual gauge theory on commutative $\mathbb{R}^{p+1}$. The new commutative coordinate of the world-volume of the BPS Dp-brane appears through the non-commutative $\mathbb{R}^2$. The extra dimension effectively disappears by the condensation of the tachyons as in the case of a soliton in a D9-brane, then the non-commutative $\mathbb{R}^2$ reduces to the commutative $\mathbb{R}^1$.

We also construct a Dp-brane from non-BPS D$(p-n)$-branes by the ABS like configuration. It is shown that the RR charges of the solitons are correct ones. The tension of the Dp-brane is shown to coincide with the expected value for $n = 1$ case. For $n > 1$, we show that the tension of the soliton computed from a natural action coincides with the expected value.

This paper is organized as follows. In section 2, we briefly review the BSFT action for non-BPS D9-brane and the construction of the soliton in it. In section 3, we construct an exact soliton, which represents a BPS Dp-brane, in a BSFT action of infinitely many non-BPS D$(p-1)$-branes. We also construct a Dp-brane from non-BPS D$(p-n)$-branes by the ABS like configuration. We confirm that the Dp-brane has correct RR charges and the tension. In section 4, we construct an exact soliton for brane-antibrane systems. Section 5 is devoted to conclusion. In appendix A we summarize the construction of Dp-brane from D$(p-2m)$-branes.

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2. Solitons in BSFT action

First we review the BSFT action for non-BPS D-brane and the construction of the soliton in it [31]. The BSFT action for the real tachyon $T(x)$ is obtained in [31] as

$$S = \tilde{T}_9 \int d^{10}x e^{-T^2/4} F\left(\frac{\alpha'}{2}\partial_\mu T \partial^\mu T\right), \tag{2.1}$$

where

$$F(x) = x \frac{4^x}{2} \frac{\Gamma(x)^2}{\Gamma(2x)} \tag{2.2}$$

and $\tilde{T}_p$ is the tension of the non-BPS Dp-brane. This action is considered to be exact if we set $T = \sum u_i x_i$ and other fields to zero. Using

$$F(x) = 1 + 2\ln 2\, x + \mathcal{O}(x^2)\,, \tag{2.3}$$

we can obtain

$$S = \tilde{T}_9 \int d^{10}x e^{-T^2/4}\left(1 + \ln 2\, \alpha' \partial_\mu T \partial^\mu T + \mathcal{O}\left((\partial_\mu T \partial^\mu T)^2\right)\right). \tag{2.4}$$

Then it was shown in [31] that the closed string vacuum corresponds to $T = \infty$ and the exact soliton which represents a BPS D8-brane is

$$T = u x_9\,, \tag{2.5}$$

with $u = \infty$ from

$$F(x) = \sqrt{\pi x} + \frac{1}{8}\sqrt{\frac{\pi}{x}} + \mathcal{O}\left(\frac{1}{x^{3/2}}\right). \tag{2.6}$$

Furthermore, the tension of the BPS D8-brane is finite and coincides with the expected values. The fluctuation around the solution was studied in [36, 37] and it was explicitly shown that the fluctuation modes live effectively on the $\mathbb{R}^9$ by the effect of the factor $e^{-T^2/4} \sim e^{-u x_9^2/4}$ which makes the fluctuations along x_9 confine in the region of a length scale $1/\sqrt{u}$. The spectrum, however, is not expected to be exact because some neglected terms in the BSFT action, for example $e^{-T^2/4}(\partial_\mu\partial_\nu T)(\partial_\mu\partial_\nu T)$, may contribute the fluctuation around the solution.

3. Construction of Dp-branes from lower-dimensional non-BPS D$(p-2m-1)$-branes

First we consider the soliton in a field theoretical toy model for the effective world-volume action of N non-BPS D-branes. The field theoretical model of a tachyon field $T(x)$ and gauge fields $A_\mu(x)$ in a D9-brane considered in [36] is

$$S = \tilde{T}_9 \int d^{10}x e^{-T^2/4}\left(1 + c_1 \alpha' \partial_\mu T \partial^\mu T + \frac{c_2}{4}(2\pi\alpha' F_{\mu\nu})^2\right), \tag{3.1}$$

where c_i are some numerical constants. Then from the T-dual relation

$$A_\mu \longrightarrow \frac{1}{\sqrt{\alpha'}}\Phi_\mu \,, \tag{3.2}$$

it is natural to consider a toy model for N non-BPS D(-1)-branes[2] which has an action

$$S = \tilde{T}_{-1}\ \mathrm{Tr}_{N\times N}\left[e^{-T^2/4} I(T,\Phi)\right], \tag{3.3}$$

where T and Φ_μ become $N\times N$ matrices and

$$I(T,\Phi) = 1 - c_1[\Phi_\mu, T]^2 - c_2\pi^2[\Phi_\mu,\Phi_\nu]^2 \,. \tag{3.4}$$

Here we have done the replacement

$$\begin{aligned} D_\mu T &= \partial_\mu T - i(A_\mu T - TA_\mu) \longrightarrow -i\frac{1}{\sqrt{\alpha'}}[\Phi_\mu, T]\,, \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\mu] \longrightarrow -i\frac{1}{\alpha'}[\Phi_\mu,\Phi_\nu]\,. \end{aligned} \tag{3.5}$$

The equations of motion for this action become

$$\begin{aligned} 0 &= \frac{\partial S}{\partial T} \sim -I(T,\Phi)Te^{-T^2/4} + 4c_1\left[\Phi_\mu, [\Phi_\mu, T]e^{-T^2/4}\right], \\ 0 &= \frac{\partial S}{\partial \Phi_\mu} \sim -c_1\left[T, [\Phi_\mu, T]e^{-T^2/4}\right], \end{aligned} \tag{3.6}$$

where we have omitted the terms containing $[\Phi_\mu,\Phi_\nu]$ or $[T,[\Phi_\mu,T]]$.

Hereafter we concentrate on the case of a system of infinite number of non-BPS D(-1)-branes, i.e. $N\to\infty$ limit, and regard the matrix of $N\times N$ as a linear operator. Then on the analogy of the solution for the case of non-BPS D9-brane (2.5), we can try to take a configuration

$$\begin{aligned} T &= 2\pi\mu\frac{1}{\sqrt{\alpha'}}\hat{p}\,, \\ \Phi_0 &= \frac{1}{2\pi\sqrt{\alpha'}}\hat{x}\,, \qquad \Phi_\mu = 0\,, \qquad (\mu = 1,\ldots,9)\,, \end{aligned} \tag{3.7}$$

where $\hat{x}$ and $\hat{p}$ are operators which satisfy

$$[\hat{x},\hat{p}] = i\alpha' \,. \tag{3.8}$$

Note that for this configuration, it is hold that $[Y_1,[Y_2,Y_3]] = 0$ where Y_i represents Φ_ν or T. Indeed, this configuration becomes a solution for the equations of motion (3.6) provided that $\mu^2 = 1/c_1$ because $[\Phi_0, T] = i\mu$ and $\partial S/\partial T \sim -T(1-c_1\mu^2)$. We note that the normalization of the operator $\hat{x}$ was taken such that the $\hat{x}$ can be regarded as the transverse coordinates of the non-BPS D(-1)-branes. We also note that the

[2] In this paper we consider the euclidean space-time for simplicity.

commutator $[\hat{x}, \hat{p}]$ is taken to depend only on α'. Actually, a solution for equations of motion for general tachyon potentials was obtained before in [35], however, as a solution of differential equation. We can easily check that the solution (3.7) solves the differential equation.

The tension of this soliton can be computed as

$$S = \tilde{T}_{-1} \operatorname{Tr} \left(e^{-\pi^2 \mu^2 \hat{p}^2/\alpha'} (1+1) \right) = 2\tilde{T}_{-1} \int dx dp \langle p | e^{-\pi^2 \mu^2 p^2/\alpha'} | x \rangle \langle x | p \rangle$$
$$= \tilde{T}_{-1} \frac{1}{\mu \sqrt{\pi^3 \alpha'}} \int dx \,, \tag{3.9}$$

where we have used the eigenstates $|x\rangle$ of operator $\hat{x}$, i.e. $\hat{x}|x\rangle = x|x\rangle$ with $\langle x|x'\rangle = \delta(x - x')$ and the eigenstates of $\hat{p}$ with $\langle p|p'\rangle = \delta(p - p')$ and $\langle x|p\rangle = \frac{1}{\sqrt{2\pi\alpha'}} e^{ipx/\alpha'}$. Although this soliton is expected to represent a BPS D0-brane, the tension of this soliton does not coincide with the BPS D0-brane tension $T_0 = \frac{\tilde{T}_{-1}}{\sqrt{2}} \frac{1}{2\pi\sqrt{\alpha'}}$ unless $\mu = 4/\sqrt{2\pi}$. This is because the action can not be regarded as an exact string field theory action.

Now we consider a solution for a BSFT action, which is derived from the action (2.1) by the T-duality,

$$S = \tilde{T}_{-1} \operatorname{Tr}_{N \times N} \left[e^{-T^2/4} \left\{ F\left(-\frac{1}{2} [\Phi_\mu, T]^2 \right) + G\left([\Phi_\mu, \Phi_\nu], [\Phi_\rho, T] \right) \right\} \right]. \tag{3.10}$$

This action may be an exact string field theory action for the configuration with

$$[T, [\Phi_\mu, T]] = [\Phi_\nu, [\Phi_\mu, T]] = [T, [\Phi_\mu, \Phi_\nu]] = [\Phi_\nu, [\Phi_\mu, \Phi_\rho]] = 0 \,, \tag{3.11}$$

which correspond to $D_\mu F_{\nu\rho} = 0$, $D_\mu D_\nu T = 0$ and $[T, D_\nu T] = 0$. Here the function F was defined by (2.2) and $G(x, y)$ is a some function which is expanded around $x = 0$ as $G(x, y) = x^2 g(y) + \mathcal{O}(x^3)$. Note that any ordering of T, $[\Phi_\mu, \Phi_\nu]$ and $[\Phi_\mu, T]$ in (3.10) can be taken to have an exact action for the configuration which satisfies (3.11).

The equations of motion for this action become

$$0 = \frac{\partial S}{\partial T} \sim -\frac{1}{2} F\left(-\frac{1}{2} [\Phi_\mu, T]^2 \right) T e^{-T^2/4} - \frac{1}{2} [\Phi_\mu, T]^2 F' \left(-\frac{1}{2} [\Phi_\mu, T]^2 \right) T e^{-T^2/4} \,,$$
$$0 = \frac{\partial S}{\partial \Phi_\mu} \sim 0 \,, \tag{3.12}$$

where we have omitted the terms containing $[\Phi_\mu, \Phi_\nu]$, $[T, [\Phi_\mu, T]]$ or $[\Phi_\nu, [\Phi_\mu, T]]$.

Then from (2.6), we can find that the configuration (3.7) is a solution for the equation of motion (3.12) provided that $\mu \to \infty$. Furthermore, the solution (3.7) is an exact solution of the string field theory. This is because the solution (3.7) satisfies $[\Phi_\mu, \Phi_\nu] = 0$ and $[[\Phi_\mu, T], H(T, \Phi_\nu)] = 0$, where $H(x, y)$ is an arbitrary function of

x, y, then (3.7) with $\mu \to \infty$ remains a solution for the equations of motion even if we include the terms omitted in the action (3.10), which vanish for the configuration satisfying (3.11).

The tension of the solution is computed as

$$S = \tilde{T}_{-1}\, \mathrm{Tr}_{N\times N}\left(e^{-\pi^2\mu^2\hat{p}^2/\alpha'} F\left(\frac{\mu^2}{2}\right)\right) = \frac{\tilde{T}_{-1}}{\sqrt{2}} \frac{1}{2\pi\sqrt{\alpha'}} \int dx\,, \tag{3.13}$$

where we have used (2.6) and

$$\mathrm{Tr}_{N\times N}\left(e^{-\pi^2\mu^2\hat{p}^2/\alpha'}\right) = \frac{1}{2\mu\sqrt{\pi^3\alpha'}} \int dx\,. \tag{3.14}$$

This indeed coincides with the tension of the D0-brane. The Chern-Simons coupling of the non-BPS branes which is relevant for this case is given by [33, 39, 40, 41] as

$$S_{\mathrm{CS}} = i\mu'\, \mathrm{Tr}_{N\times N}\left[\sqrt{\frac{\pi}{2i}} [\Phi_0, T] C_0 e^{-T^2/4}\right], \tag{3.15}$$

where μ' is a constant which can be evaluated as $\mu' = i^{-3/2}\tilde{T}_{-1}$ by the computation of the charge of the D$(p-1)$-brane soliton in non-BPS Dp-brane BSFT action. Hence we verify the charge of the soliton as

$$S_{\mathrm{CS}} = i^{3/2}\mu' \sqrt{\frac{\pi}{2}} \frac{1}{2\sqrt{\pi^3\alpha'}} \int C_0 dx, = \frac{\mu'}{\sqrt{2}} i^{3/2} \frac{1}{2\pi\sqrt{\alpha'}} \int C_0 dx = T_0 \int C_0 dx\,, \tag{3.16}$$

where T_p is a tension of a BPS Dp-brane. Note that in this computation we have not taken the limit $\mu \to \infty$ and the charge does not depend on the value of μ as expected.

Now we consider the fluctuations around the solution. First we expect that the fluctuations around the solution become effectively on the commutative $\mathbb{R}^1$. This is because the factor

$$e^{-T^2/4} \sim e^{-\frac{\pi^2}{\alpha'}\mu^2\hat{p}^2}\,, \tag{3.17}$$

with $\mu \to \infty$ would suppress fluctuations along the $\hat{p}$. Note that in the limit $\mu \to \infty$, $e^{-\frac{4\pi^2}{\alpha'}\mu^2 p^2}$ reduces to a delta function $\delta(p)$ in the commutative case and we can translate the operators to the functions by the Weyl transformation. Thus this factor coming from the tachyon condensation would effectively make the world volume of the BPS D0-brane, which is constructed as the soliton, extend along $\mathbb{R}^1$, rather than non-commutative $\mathbb{R}^2$. We note that in this construction of the D0-brane, the non-commutative $\mathbb{R}^2$ play a crucial role in order to produce the world volume coordinate from the matrices.

It is not easy, however, to calculate the effective action for the fluctuation modes since the BSFT actions used in this paper do not include the necessary terms to compute these as mentioned in section 2. Here we only restrict the fluctuations as

$$\begin{aligned} T &= 2\pi\mu \frac{1}{\sqrt{\alpha'}} \hat{p}\,, \\ \Phi_0 &= \frac{1}{2\pi\sqrt{\alpha'}} \hat{x}\,, \qquad \Phi_\mu = \delta\Phi_\mu(\hat{x})\,, \qquad (\mu = 1, \ldots, 9)\,, \end{aligned} \tag{3.18}$$

i.e. we only consider the $\hat{p}$ independent fluctuations of the transverse scalars of the D0-brane. One reason to consider these modes is that the modes are decoupled from the other modes up to quadratic fluctuation. Indeed, the action should contain the field Φ_μ as the commutator $[\Phi_\nu, \Phi_\mu]$ or $[T, \Phi_\mu]$ and these commutators are already quadratic fluctuation except

$$[T, \Phi_\mu] \sim 2\pi\mu \frac{1}{\sqrt{\alpha'}} [\hat{p}, \Phi_\mu] \sim -2i\pi\sqrt{\alpha'}\partial_{\hat{x}}(\delta\Phi_\mu(\hat{x}))\,. \tag{3.19}$$

Then inserting (3.18) and (3.19) into the action (3.10), we obtain a desired result:

$$\begin{aligned} S &= \tilde{T}_{-1}\,\mathrm{Tr}_{N\times N}\left[e^{-T^2/4}F\left(\frac{\mu^2}{2}\left(1+4\pi^2\alpha'(\partial_{\hat{x}}\delta\Phi_\mu(\hat{x}))^2\right)\right)\right] \\ &= T_0\int\sqrt{1+4\pi^2\alpha'(\partial_x\delta\Phi_\mu(x))^2}dx\,. \end{aligned} \tag{3.20}$$

Of course, we extend the construction of the higher-dimensional D-branes from the infinitely many non-BPS D(-1)-branes to the one from infinitely many non-BPS D$(2m-1)$-branes. In this case the action for the D$(2m-1)$-branes includes the gauge fields. Because the analysis is almost same as above discussions, we will consider more non-trivial generalizations.

We also construct an exact solution, which represents n BPS D0-branes:

$$\begin{aligned} T &= 2\pi\mu\frac{1}{\sqrt{\alpha'}}\left(\begin{array}{c|c|c} \hat{p} & & \\ \hline & \ddots & \\ \hline & & \hat{p} \end{array}\right), \qquad \Phi_0 = \frac{1}{2\pi\sqrt{\alpha'}}\left(\begin{array}{c|c|c} \hat{x} & & \\ \hline & \ddots & \\ \hline & & \hat{x} \end{array}\right), \\ \Phi_\mu &= \left(\begin{array}{c|c|c} C_1^\mu\mathbf{1} & & \\ \hline & \ddots & \\ \hline & & C_n^\mu\mathbf{1} \end{array}\right), \end{aligned} \tag{3.21}$$

where $\mu = 1,\ldots,9$. The constants C_i^μ correspond to positions of the i-th brane. The D0-$\overline{\text{D0}}$-brane system is obtained by changing signs of $\hat{p}$ in T, for example, $T \sim \left(\begin{array}{c|c} \hat{p} & 0 \\ \hline 0 & -\hat{p} \end{array}\right)$.

On the analogy of the ABS configuration [38], a configuration for a D$(m-1)$-brane is expected to be

$$\begin{aligned} T &= 2\pi\frac{1}{\sqrt{\alpha'}}\sum_{k=0}^{m-1}\mu_k\,\gamma_k\,\hat{p}^k\,, \\ \Phi_i &= \frac{1}{2\pi\sqrt{\alpha'}}\hat{x}^i\,, \qquad (i=0,\ldots,m-1)\,, \qquad \Phi_\mu = 0\,, \qquad (\mu = m,\ldots,9)\,, \end{aligned} \tag{3.22}$$

where γ_i are $2^{[m/2]}\times 2^{[m/2]}$ hermitean gamma matrices which satisfy $\{\gamma_i,\gamma_j\} = 2\delta_{ij}$ and $[\hat{x}^i, \hat{p}^j] = i\alpha'\delta_{ij}$. The D-brane charges of this configuration are expected to vanish for m = even, i.e. the non-BPS D$(m-1)$-brane. Indeed, we can see that the RR charges of the soliton coincide with the expected values. The Chern-Simons coupling for the several non-BPS D(−1)-branes was obtained by the BSFT [33] as,

$$S_{\rm CS} = \tilde{T}_{-1} i^{-1/2}\,{\rm SymTr}\left[e^{-\frac{1}{4}T^2} e^{-2\pi i\ \mathbf{i}_\Phi \mathbf{i}_\Phi + \sqrt{\pi/2i}[\mathbf{i}_\Phi, T]} \wedge C\right]\Big|_{dx^i=0}\,, \tag{3.23}$$

where the SymTr denotes a symmetric trace with respect to $[\Phi_\mu,\Phi_\nu]$, $[\Phi_\mu,T]$ and T^2 and $\mathbf{i}_\Phi$ denotes the interior product by Φ:

$$\begin{aligned} C &= \frac{1}{r!} C_{\nu_1,\nu_2,\ldots,\nu_r} dx^{\nu_1} dx^{\nu_2}\cdots dx^{\nu_r} \\ \longrightarrow \mathbf{i}_\Phi C &= \frac{1}{(r-1)!}\Phi^i C_{i,\nu_2,\ldots,\nu_r} dx^{\nu_2}\cdots dx^{\nu_p}\,. \end{aligned} \tag{3.24}$$

Here $[\quad]|_{dx^i=0}$ means that we should set $dx^i = 0$ and keep 0-form only. Thus we can obtain the RR charges of the soliton as

$$C_{\rm CS} = i\frac{1}{\sqrt{2}}\mu'(-i)^{[m/2]} i^{m/2}\left(\frac{1}{2\pi\sqrt{\alpha'}}\right)^m \int C^{10-m} = T_{m-1}\int C^{10-m}\,, \tag{3.25}$$

for $m =$ odd. Here $\mu' = \tilde{T}_{-1} i^{-3/2}$ and we have used ${\rm Tr}_\gamma(\gamma_1\cdots\gamma_{2n+1}) = 2^n(-i)^n$.

It is difficult, however, to show that the configuration is a solution because $[T,[\Phi_j,T]] = i2\pi\frac{1}{\sqrt{\alpha'}}\mu_j\sum_{k=0}^{m-1}\mu_k\,[\gamma_k,\gamma_j]\,\hat{p}^k \neq 0$. Instead of showing this, we consider another action, which seem to be natural, and compute the tension using this action. To this end, let us remember that for the configuration $T = \sum_{k=0}^{m-1}\mu_k\,\gamma_k\,x^k$, the BSFT action for $2^{[m/2]}$ non-BPS D9-branes becomes [31]

$$\begin{aligned} S' &= \tilde{T}_9\, 2^{[m/2]}\int d^{10}x\, e^{-T^2/4}\prod_{k=0}^{m-1}\left\{F\left(\frac{\alpha'}{2}\mu_k^2\right)\right\} \\ &= \tilde{T}_9\, 2^{[m/2]}\int d^{10}x\, e^{-T^2/4}\prod_{k=0}^{m-1}\left(1 + (\ln 2)\alpha'\mu_k^2 + \beta\alpha'^2\mu_k^4 + \cdots\right) \\ &= \tilde{T}_9\, 2^{[m/2]}\int d^{10}x\, e^{-T^2/4}\Bigg(1 + (\ln 2)\alpha'\sum_{k=0}^{m-1}(\mu_k^2) + \beta\alpha'^2\sum_{k=0}^{m-1}(\mu_k^4) + \\ &\qquad\qquad + \alpha'^2\sum_{k,l=0,\,k>l}^{m-1}(\ln 2)^2\mu_k^2\mu_l^2 + \cdots\Bigg) \\ &= \tilde{T}_9\int d^{10}x\, e^{-T^2/4}\,{\rm Tr}_\gamma\Bigg[1 + (\ln 2)\alpha'(D_lT)^2 - \frac{1}{2}(\ln 2)^2\alpha'^2(D_lTD_mT)^2 + \\ &\qquad\qquad + \left(\beta + \frac{1}{2}(\ln 2)^2\right)\alpha'^2(D_lT)^2(D_mT)^2 + \cdots\Bigg]\,, \end{aligned} \tag{3.26}$$

where β is a numerical constant and we have used ${\rm Tr}_\gamma(\gamma_k\gamma_l\gamma_k\gamma_l) = 2({\rm Tr}\,1)(2\delta_{kl}-1)$. Then the BSFT action for the $N\times 2^{[m/2]}$ non-BPS D(−1)-branes is expected to be

$$\begin{aligned} S = \tilde{T}_{-1}\,{\rm Tr}_\gamma\ {\rm Tr}_{N\times N}\Bigg[e^{-T^2/4}\Bigg(&1 - (\ln 2)[\Phi_\mu,T]^2 - \frac{1}{2}(\ln 2)^2\left([\Phi_\mu,T][\Phi_\nu,T]\right)^2 + \\ &+ \left(\beta + \frac{1}{2}(\ln 2)^2\right)[\Phi_\mu,T]^2[\Phi_\nu,T]^2 + \cdots\Bigg)\Bigg]\,. \end{aligned} \tag{3.27}$$

Note that this action may be consistent with the action (3.10) with $N \to 2^{[m/2]}N$ since (3.27) may coincides with (3.10) for $[[\Phi_\mu, T], [\Phi_\nu, T]] = 0$. The action (3.27) is evaluated for the soliton as

$$S = \tilde{T}_{-1} 2^{[m/2]} \ \mathrm{Tr}_{N\times N} \left[\prod_{k=0}^{m-1} e^{-\pi^2 \mu_k^2 \hat{p}_k^2/\alpha'} F\left(\frac{\mu_k^2}{2}\right) \right] = T'_{m-1} \int dx^0 \cdots dx^{m-1} , \quad (3.28)$$

where

$$T'_{m-1} = \tilde{T}_{-1} 2^{[m/2]-m} \left(\frac{1}{\sqrt{2\alpha'}\pi} \right)^m . \quad (3.29)$$

Therefore we have found that the T'_{m-1} is coincide with the D$(m-1)$-brane tension $T^{\mathrm{non-BPS}}_{m-1} = (\frac{1}{2\pi\sqrt{\alpha'}})^m \tilde{T}_{-1}$ for $m =$ even, or $T^{\mathrm{BPS}}_{m-1} = (\frac{1}{2\pi\sqrt{\alpha'}})^m \frac{\tilde{T}_{-1}}{\sqrt{2}}$ for $m =$ odd.

Another exact solution corresponding to BPS D$(2m)$-brane is an analogue of the usual construction of the non-commutative brane as in [35]:

$$\begin{aligned} T &= 2\pi\mu \frac{1}{\sqrt{\alpha'}} \hat{p} \,, \qquad \Phi_0 = \frac{1}{2\pi\sqrt{\alpha'}} \hat{x} \,, \\ \Phi_{2i-1} &= \frac{1}{2\pi\sqrt{\alpha'}} \hat{x}_i \,, \qquad \Phi_{2i} = \frac{1}{2\pi\sqrt{\alpha'}} \hat{p}_i \,, \qquad (i = 1, \ldots, m) \,, \\ \Phi_\mu &= 0 \,, \qquad (\mu = 2m+1, \ldots, 9) \,, \end{aligned} \quad (3.30)$$

where $\hat{x}_i$ and $\hat{p}_i$ are operators which satisfy

$$[\hat{x}_i, \hat{p}_j] = i\delta_{ij}\alpha' . \quad (3.31)$$

As the usual construction of the non-commutative D-branes, this solution represents the BPS D$(2m)$-brane with the lower-dimensional D-brane charges, or background constant NS-NS B-field. This can be verified by the computation of the RR charges from the Chern-Simons couplings (3.23). We can also construct the exact soliton corresponding to a non-BPS D$(2m-1)$-brane with background constant NS-NS B-field as

$$\begin{aligned} \Phi_{2i-2} &= \frac{1}{2\pi\sqrt{\alpha'}} \hat{x}_i \,, \qquad \Phi_{2i-1} = \frac{1}{2\pi\sqrt{\alpha'}} \hat{p}_i \,, \qquad (i = 1, \ldots, m) \,, \\ \Phi_\mu &= 0 \,, \qquad (\mu = 2m, \ldots, 9) \,, \qquad T = 0 \,. \end{aligned} \quad (3.32)$$

In appendix A, we will discuss these cases further.

Finally, we comment on the combinations of the above constructions. If we notice that the direct sum of the above configurations is also an exact solution of the equations of motion, we can easily construct the composite system of any number of Dq-branes and D$(q+m)$-branes. Therefore it is an advantage of the techniques in this paper that we can easily construct the system of D-branes with different dimensions.

4. Brane-antibrane System

For D9-$\bar{\text{D}}$9 system, the BSFT action have been obtained [32, 33] as

$$
\begin{aligned}
S' &= 2T_9 \int d^{10}x\, e^{-T\bar{T}} F\left(\frac{\alpha'}{2}\mu_1^2\right) F\left(\frac{\alpha'}{2}\mu_2^2\right) \\
&= 2T_9 \int d^{10}x\, e^{-T\bar{T}} \prod_{k=1}^{2} \left(1 + (\ln 2)\alpha'\mu_k^2 + \beta\alpha'^2\mu_k^4 + \cdots\right) \\
&= 2T_9 \int d^{10}x\, e^{-T\bar{T}} \left(1 + (\ln 2)\alpha'(\mu_1^2+\mu_2^2) + \beta\alpha'^2(\mu_1^4+\mu_2^4) + \alpha'^2(\ln 2)^2\mu_1^2\mu_2^2 + \cdots\right) \\
&= 2T_9 \int d^{10}x\, e^{-T\bar{T}} \left[1 + 4(\ln 2)\alpha'\partial_l T\partial_l\bar{T} + 4\left(2\beta - (\ln 2)^2\right)\alpha'^2(\partial_l T\partial_l\bar{T})^2 + \right. \\
&\qquad\qquad \left. + 4\left(2\beta + (\ln 2)^2\right)\alpha'^2(\partial_l T\partial_m\bar{T})^2 + \cdots\right], \qquad (4.1)
\end{aligned}
$$

for the configuration

$$
T = \frac{1}{2}(i\mu_1 x_1 + \mu_2 x_2), \qquad A_\mu^{(i)} = 0, \qquad i = 1, 2. \qquad (4.2)
$$

Thus the action for N D(-1)-$\bar{\text{D}}(-1)$ pairs is expected to be

$$
\begin{aligned}
S = T_{-1}\,\mathrm{Tr}_{N\times N} \Big[e^{-T\bar{T}} \Big\{ & 1 + 4(\ln 2)\alpha' \left(\Phi_l^{(1)}T - T\Phi_l^{(2)}\right)\left(\bar{T}\Phi_l^{(1)} - \Phi_l^{(2)}\bar{T}\right) + \\
& + 4\left(\beta - (\ln 2)^2\right)\alpha'^2 \left(\Phi_l^{(1)}T - T\Phi_l^{(2)}\right)\left(\bar{T}\Phi_l^{(1)} - \Phi_l^{(2)}\bar{T}\right) \times \\
& \times \left(\Phi_m^{(1)}T - T\Phi_m^{(2)}\right)\left(\bar{T}\Phi_m^{(1)} - \Phi_m^{(2)}\bar{T}\right) + \\
& + 4\left(2\beta + (\ln 2)^2\right)\alpha'^2 \left(\Phi_l^{(1)}T - T\Phi_l^{(2)}\right)\left(\bar{T}\Phi_m^{(1)} - \Phi_m^{(2)}\bar{T}\right) \times \\
& \times \left(\Phi_l^{(1)}T - T\Phi_l^{(2)}\right)\left(\bar{T}\Phi_m^{(1)} - \Phi_m^{(2)}\bar{T}\right) + \cdots \Big\} + \qquad (4.3) \\
+ e^{-\bar{T}T} \Big\{ & 1 + 4(\ln 2)\alpha' \left(\bar{T}\Phi_l^{(1)} - \Phi_l^{(2)}\bar{T}\right)\left(\Phi_l^{(1)}T - T\Phi_l^{(2)}\right) \cdots \Big\} \Big],
\end{aligned}
$$

where we have omitted T-independent terms and higher commutator terms. For this action we can trace the construction of the higher-dimensional D-branes considered in the section 3.

For example, we can construct an exact solution for the infinitely many D(-1)-$\bar{\text{D}}(-1)$ pairs as

$$
\begin{aligned}
T &= \frac{\pi}{\sqrt{\alpha'}}(i\mu_1\hat{p}_1 + \mu_2\hat{p}_2), \qquad \Phi_0^{(1)} = \Phi_0^{(2)} = \frac{1}{2\pi\sqrt{\alpha'}}\hat{x}_1, \\
\Phi_1^{(1)} &= \Phi_1^{(2)} = \frac{1}{2\pi\sqrt{\alpha'}}\hat{x}_2, \qquad \text{other fields} = 0, \qquad (4.4)
\end{aligned}
$$

which corresponds to a BPS D1-brane. A solution which is corresponds to a D$(m-1)$-

brane can be also constructed for $2^{[(m-1)/2]} \times N$ D(-1)-$\bar{\mathrm{D}}(-1)$ pairs as

$$T = i\pi \frac{1}{\sqrt{\alpha'}} \sum_{k=0}^{m-1} \mu_k \, \Gamma_k \, \hat{p}^k \,,$$
$$\Phi_i^{(1)} = \Phi_i^{(2)} = \frac{1}{2\pi\sqrt{\alpha'}} \hat{x}^i \,, \qquad (i = 0, \ldots, m-1) \,, \qquad \text{other fields} = 0 \,, \quad (4.5)$$

where Γ^k denote $2^{[\frac{m-1}{2}]} \times 2^{[\frac{m-1}{2}]}$ hermitean gamma-matrices in $m-1$ dimension for $k = 1, \ldots, m-1$ and $\Gamma^0 = i\mathbf{1}$. If we define a shift operator S such as $S^\dagger S = 1$ and $SS^\dagger = 1 - P$, where P is a projection operator with $\dim(\mathrm{Ker}\, P) = N_0$, a configuration $T = \mu S$ with $\mu \to \infty$ is an exact solution corresponding to N_0 D(-1)-branes.[3]

5. Conclusions

In this paper we have constructed an exact soliton, which represents a BPS Dp-brane, in the effective world-volume theory of infinitely many non-BPS D$(p-1)$-branes. We have argued that the world-volume theory of the BPS Dp-brane constructed in this way becomes usual gauge theory on commutative $\mathbb{R}^{p+1}$. The new commutative coordinate of the world-volume of the BPS Dp-brane is appeared through the non-commutative $\mathbb{R}^2$. The extra dimension effectively disappears by the condensation of the tachyons as in the case of a soliton in a D9-brane. We have also constructed a Dp-brane from non-BPS D$(p-n)$-branes by the ABS like configuration. It has been shown that the RR charges and tensions of the solitons are correct ones.

It is very interesting to study the fluctuations around the exact solitons including the fermion fields to the action as discussed in [37]. In order to consider the fluctuations properly, we probably have to calculate the BSFT action including the higher derivative terms.

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A. Dp-brane from D$(p-2m)$-branes

First we consider the construction of BPS Dp-brane from infinitely many BPS D$(p-2)$-branes following [1]–[6]. For simplicity we consider the construction from BPS D(-1)-branes only. The bosonic part of the effective action for the N BPS D(-1)-

[3]This solution was suggested by S. Sugimoto.

branes may be written as

$$S_N = T_{-1}\ \mathrm{SymTr}_{N\times N}\left[\sqrt{\det\left(g_{\mu\nu} - i2\pi[\Phi_\mu, \Phi_\nu]\right)}\right], \tag{A.1}$$

where we have omitted the terms which contain two or more commutators and T_{-1} is the tension of a BPS D(−1)-brane. It can be easily seen that the equations of motion for this action with the limit $N \to \infty$ are satisfied by a configuration

$$\Phi^i = \frac{1}{2\pi\sqrt{\alpha'}}\hat{x}^i, \qquad (i = 1, 2, \ldots, 2m), \tag{A.2}$$

where

$$[\hat{x}^j, \hat{x}^k] = i\theta^{jk}, \qquad (j, k = 1, 2, \ldots, 2m), \tag{A.3}$$

Note that there are moduli θ^{jk}. Here we take $\theta^{(2j-1),2j} = -\theta^{2j,(2j-1)} \equiv \theta_j$ and the other components vanish.

Because we expect that the full effective action contains Φ_μ only as a commutator $[\Phi_\mu, \Phi_\nu]$, the omitted terms in (A.1) can be ignored for the configuration (A.2) as far as the equations of motion is concerned. Therefore we find that (A.2) is the exact solution of the full string field theory and corresponds to a D$(2m-1)$-brane with $B_{jk} = (1/\theta)_{jk}$ [9]. Here following [11], we have chosen $\Phi_{ij} = -B_{ij}$, where Φ_{ij} represent some degree of freedom to write the non-commutative action [9].

The tension of the solution is evaluated as

$$\begin{aligned} S_N &= T_{-1}\int dx_1 \int dx_2 \cdots \int dx_{2m-1}\int dx_{2m}\sqrt{\prod_{i=1}^{m}\left(1+\frac{\theta_i^2}{4\pi^2\alpha'^2}\right)}\prod_{j=1}^{m}\frac{1}{2\pi\theta_j} \\ &= T_{-1}\left(\frac{1}{2\pi\sqrt{\alpha'}}\right)^{2m}\prod_{k=1}^{m}\left(\sqrt{1+\left(\frac{2\pi\alpha'}{\theta_k}\right)^2}\int dx_{2k-1}\int dx_{2k}\right). \end{aligned} \tag{A.4}$$

This coincides with the tension of a D$(2m-1)$-brane since the world-volume theory of the D$(2m-1)$-brane with non-vanishing B-field becomes

$$S_{2m-1} = T_{(2m-1)}\left(\prod_{k=1}^{m}\int dx_{2k-1}\int dx_{2k}\right)\sqrt{\det\left(g_{ij} + 2\pi\alpha' B_{ij}\right)}, \tag{A.5}$$

which agrees with (A.4).

If we denote fluctuations around the solution as

$$\Phi^i = \frac{1}{2\pi\sqrt{\alpha'}}\left(\hat{x}^i - \hat{A}_j\theta^{ji}\right), \qquad (i = 1, \ldots, 2m), \tag{A.6}$$

we find [10, 11]

$$[\Phi^i, \Phi^j] = -i\frac{1}{4\pi^2\alpha'}\left(\theta^{ij} - \theta^{ik}\theta^{lj}\hat{F}_{kl}\right) = i\frac{1}{4\pi^2\alpha'}\theta^{ik}\theta^{lj}\left(\hat{F}_{kl} + \Phi_{kl}\right), \tag{A.7}$$

where $\hat{F}_{kl} = \partial_k \hat{A}_l - \partial_l \hat{A}_k - i[\hat{A}_k, \hat{A}_l]$. We can also show that

$$\begin{aligned}(-i)[\Phi^{i_1}, \Phi_{i_2}](-i)[\Phi^{i_2}, \Phi_{i_3}] \cdots (-i)[\Phi^{i_{2n-1}}, \Phi_{i_{2n}}](-i)[\Phi^{i_{2n}}, \Phi_{i_1}] = \\ = \mathrm{Tr}((G^{-1}(F + \Phi))^{2n}) \,, \end{aligned} \tag{A.8}$$

where $G_{ij} = -(2\pi\alpha')^2 (B^2)_{ij}$ is the open string metric [9]. Using this, we can show that an action for the fluctuations around the solution indeed becomes the non-commutative effective world-volume theory of the D$(2m-1)$-brane

$$S_N = T_{2m-1} \frac{g_s}{G_s} \sqrt{\det(2\pi\theta^{ij})}\ \mathrm{SymTr}_{N\times N} \left(\sqrt{\det(G_{ij} + 2\pi\alpha'(\hat{F}_{ij} + \Phi_{ij}))} \right) , \tag{A.9}$$

where $G_s = g_s \sqrt{\det(2\pi\alpha' B_{ij})}$ is the open string coupling constant [9]. Therefore the action (A.1) and (A.9) are the same action with the different backgrounds, which satisfy the equations of motion.

Now we consider the omitted terms in (A.1), which are higher commutator terms. In principle, we can determine these terms from the non-abelian effective world-volume action of D9-branes by a replacement $2\pi\alpha' F_{ij} \to -i[\Phi_i, \Phi_j]$ and $(2\pi\alpha')^{\frac{m}{2}+1} D_{i_1} \cdots D_{i_m}(F_{kl}) \to -i^{m+1}[\Phi_{i_1}, [\ldots, [\Phi_{i_m}, [\Phi_k, \Phi_l]]]]$. From (A.6), we can regard $i\Phi_i$ by $\sqrt{2\pi\alpha'}\hat{D}_j = \sqrt{2\pi\alpha'}(\hat{\partial}_j + i\hat{A}_j)$ with the open string metric in the non-commutative gauge theory picture where $[\hat{\partial}_j, f(\hat{x}, \hat{p})] = \partial_j f(\hat{x}, \hat{p})$. Therefore we find that the full effective action for the fluctuations around the solution has same form as the non-abelian effective world-volume action of D9-branes, as expected.

The D-brane charges can be also evaluated from the Chern-Simons term [42]

$$S_{\mathrm{CS}} = T_{-1}\, \mathrm{SymTr}_{N\times N} \left(e^{-2\pi i\, \mathbf{i}_\Phi \mathbf{i}_\Phi} \wedge C \right) , \tag{A.10}$$

which may be obtained from the Chern-Simons term for BPS D9-brane $S_{\mathrm{CS}}^{D9} = T_9 \int d^{10}x e^{2\pi\alpha' F} \wedge C$. Substituting the solution into (A.10), it is obtained that

$$S_{\mathrm{CS}} = T_{2m-1} \int d^{2m}x\, e^{2\pi\alpha' B} \wedge C \,. \tag{A.11}$$

Now we consider the construction of non-BPS D$(2m-1)$-brane from infinitely many non-BPS D(-1)-branes. The action for these was given by (3.10). The solution for this case was given by (3.32) and the above analysis for the BPS case can be applied without essential modifications. In particular, we can see that the non-commutative gauge theory action and the matrix action (3.10) are the same action with the different backgrounds, which satisfy the equations of motion. In [27, 28] the non-commutative soliton corresponding to N_0 non-BPS D(-1)-branes was constructed in the non-commutative world-volume theory of D$(2m-1)$-brane. It can be identified in the action (3.10) as an almost trivial solution

$$T = T_c(1 - P) \,, \qquad \Phi_\mu = 0 \,, \tag{A.12}$$

where $T_c = \infty$ and P is a projector which obeys $\dim(\mathrm{Ker}(1-P)) = N_0$.

References

[1] T. Banks, W. Fischler, S.H. Shenker and L. Susskind, *M-theory as a matrix model: a conjecture*, *Phys. Rev.* **D 55** (1997) 5112 [hep-th/9610043].

[2] N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, *A large-N reduced model as superstring*, *Nucl. Phys.* **B 498** (1997) 467 [hep-th/9612115].

[3] T. Banks, N. Seiberg and S. Shenker, *Branes from matrices*, *Nucl. Phys.* **B 490** (1997) 91 [hep-th/9612157].

[4] P.K. Townsend, *D-branes from M-branes*, *Phys. Lett.* **B 373** (1996) 68 [hep-th/9512062].

[5] B. de Wit, J. Hoppe and H. Nicolai, *On the quantum mechanics of supermembranes*, *Nucl. Phys.* **B 305** (1988) 545.

[6] N. Ishibashi, *p-branes from $(p-2)$-branes in the bosonic string theory*, *Nucl. Phys.* **B 539** (1999) 107 [hep-th/9804163].

[7] A. Connes, M.R. Douglas and A. Schwarz, *Noncommutative geometry and matrix theory: compactification on tori*, *J. High Energy Phys.* **02** (1998) 003 [hep-th/9711162].

[8] M.R. Douglas and C. Hull, *D-branes and the noncommutative torus*, *J. High Energy Phys.* **02** (1998) 008 [hep-th/9711165].

[9] N. Seiberg and E. Witten, *String theory and noncommutative geometry*, *J. High Energy Phys.* **09** (1999) 032 [hep-th/9908142].

[10] H. Aoki, N. Ishibashi, S. Iso, H. Kawai, Y. Kitazawa and T. Tada, *Noncommutative Yang-Mills in IIB matrix model*, *Nucl. Phys.* **B 565** (2000) 176 [hep-th/9908141].

[11] N. Seiberg, *A note on background independence in noncommutative gauge theories, matrix model and tachyon condensation*, *J. High Energy Phys.* **09** (2000) 003 [hep-th/0008013].

[12] For a review, see A. Sen, *Non-BPS states and branes in string theory*, hep-th/9904207.

[13] E. Witten, *D-branes and K-theory*, *J. High Energy Phys.* **12** (1998) 019 hep-th/9810188.

[14] V.A. Kostelecky and S. Samuel, *The static tachyon potential in the open bosonic string theory*, *Phys. Lett.* **B 207** (1988) 169; *On a nonperturbative vacuum for the open bosonic string*, *Nucl. Phys.* **B 336** (1990) 263;
A. Sen and B. Zwiebach, *Tachyon condensation in string field theory*, *J. High Energy Phys.* **03** (2000) 002 [hep-th/9912249];
N. Berkovits, A. Sen and B. Zwiebach, *Tachyon condensation in superstring field theory*, *Nucl. Phys.* **B 587** (2000) 147 [hep-th/0002211].

[15] A. Sen, SO(32) *spinors of type I and other solitons on brane-antibrane pair*, *J. High Energy Phys.* **09** (1998) 023 [hep-th/9808141].

[16] A. Sen, *BPS D-branes on non-supersymmetric cycles*, *J. High Energy Phys.* **12** (1998) 021 [hep-th/9812031].

[17] O. Bergman and M.R. Gaberdiel, *Stable non-BPS D-particles*, *Phys. Lett.* **B 441** (1998) 133 [hep-th/9806155].

[18] A. Sen, *Type-I D-particle and its interactions*, *J. High Energy Phys.* **10** (1998) 021 [hep-th/9809111].

[19] M. Frau, L. Gallot, A. Lerda and P. Strigazzi, *Stable non-BPS D-branes in type-I string theory*, *Nucl. Phys.* **B 564** (2000) 60 [hep-th/9903123].

[20] M.B. Green, *Pointlike states for type-IIB superstrings*, *Phys. Lett.* **B 329** (1994) 435 [hep-th/9403040].

[21] T. Banks and L. Susskind, *Brane-antibrane forces*, hep-th/9511194.

[22] A. Sen, *Tachyon condensation on the brane antibrane system*, *J. High Energy Phys.* **08** (1998) 012 [hep-th/9805170].

[23] R. Gopakumar, S. Minwalla and A. Strominger, *Noncommutative solitons*, *J. High Energy Phys.* **05** (2000) 020 [hep-th/0003160].

[24] K. Dasgupta, S. Mukhi and G. Rajesh, *Noncommutative tachyons*, *J. High Energy Phys.* **06** (2000) 022 [hep-th/0005006].

[25] J.A. Harvey, P. Kraus, F. Larsen and E.J. Martinec, *D-branes and strings as non-commutative solitons*, *J. High Energy Phys.* **07** (2000) 042 [hep-th/0005031].

[26] E. Witten, *Noncommutative tachyons and string field theory*, hep-th/0006071.

[27] M. Aganagic, R. Gopakumar, S. Minwalla and A. Strominger, *Unstable solitons in noncommutative gauge theory*, *J. High Energy Phys.* **04** (2001) 001 [hep-th/0009142].

[28] J.A. Harvey, P. Kraus and F. Larsen, *Exact noncommutative solitons*, *J. High Energy Phys.* **12** (2000) 024 [hep-th/0010060].

[29] A.A. Gerasimov and S.L. Shatashvili, *On exact tachyon potential in open string field theory*, *J. High Energy Phys.* **10** (2000) 034 [hep-th/0009103].

[30] D. Kutasov, M. Marino and G. Moore, *Some exact results on tachyon condensation in string field theory*, *J. High Energy Phys.* **10** (2000) 045 [hep-th/0009148].

[31] D. Kutasov, M. Marino and G. Moore, *Remarks on tachyon condensation in superstring field theory*, hep-th/0010108.

[32] P. Kraus and F. Larsen, *Boundary string field theory of the* $\mathrm{D}\bar{\mathrm{D}}$ *system*, *Phys. Rev.* **D 63** (2001) 106004 [hep-th/0012198].

[33] T. Takayanagi, S. Terashima and T. Uesugi, *Brane-antibrane action from boundary string field theory*, *J. High Energy Phys.* **03** (2001) 019 [hep-th/0012210].

[34] E. Witten, *On background independent open string field theory*, *Phys. Rev.* **D 46** (1992) 5467 [hep-th/9208027]; *Some computations in background independent off-shell string theory*, *Phys. Rev.* **D 47** (1993) 3405 [hep-th/9210065].

[35] J. Kluson, *D-branes from N non-BPS D0-branes*, *J. High Energy Phys.* **11** (2000) 016 [hep-th/0009189].

[36] J.A. Minahan and B. Zwiebach, *Effective tachyon dynamics in superstring theory*, *J. High Energy Phys.* **03** (2001) 038 [hep-th/0009246].

[37] J.A. Minahan and B. Zwiebach, *Gauge fields and fermions in tachyon effective field theories*, *J. High Energy Phys.* **02** (2001) 034 [hep-th/0011226].

[38] M.F. Atiyah, R. Bott and A. Shapiro, *Clifford modules*, *Topology* **3** (1964) 3.

[39] M. Billó, B. Craps and F. Roose, *Ramond-Ramond couplings of non-BPS D-branes*, *J. High Energy Phys.* **06** (1999) 033 [hep-th/9905157].

[40] B. Janssen and P. Meessen, *A non-abelian chern-simons term for non-BPS D-branes*, hep-th/0009025.

[41] S. Mukhi and N.V. Suryanarayana, *Chern-simons terms on noncommutative branes*, *J. High Energy Phys.* **11** (2000) 006 [hep-th/0009101].

[42] R.C. Myers, *Dielectric-branes*, *J. High Energy Phys.* **12** (1999) 022 [hep-th/9910053].